Newtonian Flow Over Oscillating Two-Dimensional Airfoils at Moderate or Large Incidence

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This paper considers the problem of pitching oscillating sharp-edged, two-dimensional thin airfoils in Newtonian flow at moderate or large angles of attack. By extending and perturbing the steady flow solution past the compression surface of such airfoils, simple formulas for the unsteady surface pressure and the aerodynamic derivative are found. It is shown that moderate angles of attack have a destabilizing effect for certain pivoting positions, which increases with increasing concavity. On the other hand, pitching oscillations of concave or convex airfoils at large angles of attack are always dynamically stable.

Nomenclature

a	=	constant, Eq. (9)
C_m, $-C_{m_\theta}$, $-C_{m_{\dot\theta}}$	=	coefficient of pitching moment, in-phase and out-of-phase components of the damping derivative
$\bar F,\bar F_0$	=	unsteady and steady physical equation of airfoil
F_0	=	steady nondimensional equation of airfoil, Eq. (7)
$\bar h$	=	distance between pitching axis and vertex
i	=	imaginary unity
k	=	reduced frequency
$\bar\ell,\ell$	=	airfoil length, subscript denoting compression surface
M_∞,M_h	=	freestream Mach number, pitching moment
N	=	constant, Eq. (10)
$P_\infty,\bar p,p$	=	freestream pressure, physical and steady perturbation pressures
P_0,P_1	=	unsteady perturbation pressures
P_{0r},kP_{0i}	=	real and imaginary parts of P_0 at airfoil surface
P_{1r},kP_{1i}	=	real and imaginary parts of P_1 at airfoil surface
$\bar q$	=	velocity vector
$R_1,\bar S,S$	=	unsteady perturbation density, physical shock functions
S_0,S_1	=	steady and unsteady shock functions
$\bar t,t$	=	physical and nondimensional time variables
$U_\infty,\bar u$	=	freestream velocity and physical velocity in $\bar x$ direction
u,U_1	=	steady and unsteady perturbation velocities in $\bar x$ direction
V_0,V_1	=	unsteady perturbation velocities in $\bar y$ direction
$\bar v,v$	=	physical and steady perturbation velocities in $\bar y$ direction
$\bar x,\bar y,x,y$	=	physical and nondimensional coordinates, Eq. (7)
α	=	angle of attack
γ	=	ratio of specific heats
ϵ	=	small perturbation parameter
λ_0	=	amplitude of pitching oscillations
ω	=	circular frequency
$\rho_\infty,\bar\rho,\rho$	=	freestream, physical, and steady perturbation densities
ϕ,ψ	=	functions of $y-F_0(x)$
∇	=	differential operator

Received Jan. 25, 1988; revision received June 27, 1988.

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I. Introduction

UNSTEADY supersonic flow past two-dimensional airfoils is receiving great attention in current research. This is due to the demand for information on the unsteady aerodynamic forces and the dynamic stability of high-performance modern aircraft and re-entry space vehicles such as the Space Shuttle. In view of the fact that experimental work is almost nonexistent at high supersonic speeds, theoretical study of the problem becomes the main source of information.

Pitching oscillation of symmetric pointed-nose wedges with attached shock waves has been studied in great detail. In particular, the case of small-amplitude pitching oscillation of wedges was studied by Appleton,[1] McIntosh,[2] and Hui,[3,4] among others.[5-7] The case of large-amplitude slow oscillation was considered by Kuiken[8] and Hui,[9] while viscous effects were treated by Orlik-Ruckemann[10] and Hui and East.[11] Recently, pitching oscillations of two-dimensional thin airfoils with pointed noses and curved surfaces at small angles of attack were considered by Hemdan[12,13] at moderate supersonic speeds and in the Newtonian limit. By extending and perturbing (for small-amplitude pitching oscillations) a recent formulation of the hypersonic small-disturbance theory found by the present author,[14] Hemdan derived approximate equations for oscillating two-dimensional thin airfoils at small angles of attack. Unlike the hypersonic small-disturbance theory, this formulation and its extension to unsteady flow are both valid for a wide range of supersonic flow extending from moderate speeds to Newtonian flow. In Ref. 12, Hemdan has given simple closed-form formulas for the stability derivative of any airfoil with curved surfaces and has shown that the pitching oscillations may become dynamically unstable, especially for pivot positions around the trailing edge of the airfoil. In Ref. 13, a zero-order theory for unsteady Newtonian flow at small angles of attack is given. It shows that the curvature tends to decrease the damping derivative for certain pivot positions, depending on whether the surfaces are concave or convex, and that the wing may become dynamically unstable.

In three dimensions, pitching oscillations of flat delta wings and delta wings with conical thickness in Newtonian flow and with a shock wave detached from the leading edges but attached to the apex of the wing were studied by Hemdan and Hui[15,16] by extending Messiter's[17] steady thin shock layer theory for flat delta wings. Also, the case of a shock wave attached all along the leading edges of a flat delta wing was considered by Liu and Hui[18] by extending Hui's[19] steady flow theory.

In a series of recent interesting papers, Hui and his co-workers[20-25] have considered high Mach number flows past two-dimensional airfoils including concave, convex, and power law bodies and three-dimensional wings and aircraft performing small-amplitude pitching oscillations. Plunging motion also was considered in Ref. 25. The above investigators have developed an unsteady Newton-Busemann theory for high Mach number flows. They also gave analytic formulas for the damping derivative and characterized the stability of wings. Barron and Mandl[26] also have studied unsteady Newtonian flow past pitching power law bodies.

In contrast to the case of small angles of attack, pitching oscillations of thin airfoils with curved surfaces at moderate or large angles of attack have received very little attention. Practically speaking, however, the case of large angles of attack is the one we need to have more information about. This is most evident from Orlik-Ruckemann's[27] survey about aircraft needs and capabilities. In this survey, Orlik-Ruckemann emphasized the need for information about the damping derivative in pitching oscillations at moderate or large angles of attack. Recently, Hemdan[28] has given a new analytical solution of the hypersonic thin airfoil problem at moderate or large angles of attack in the Newtonian limit. This solution will be outlined briefly in Sec. II.

The purpose of this work is to study the unsteady flow past pitching oscillating thin airfoils in Newtonian flow at moderate or large angles of attack by perturbing the above-mentioned steady flow theory to unsteady flow. The same airfoil considered in Ref. 28 now will be assumed to make small-amplitude pitching oscillations. In Sec. II the perturbation method is used to derive approximate equations for unsteady flow past such airfoils. In Sec. III, a closed-form solution to the approximate equations derived in Sec. II is given, and in Sec. IV, a simple formula for the unsteady surface pressure is derived and used to find the damping derivative. We also present results and characterize the stability of oscillations in this section.

II. Perturbation Theory

Consider a two-dimensional thin airfoil of length $\bar{\ell}$ (physical quantities will be denoted by a bar) and sharp leading edge to be performing small-amplitude pitching oscillations about an axis at a distance $\bar{h}$ from the leading edge (see Fig. 1). Let the freestream be at a moderate or large angle of attack α, and let γ and M_∞ denote, respectively, the ratio of the specific heats of the gas and the freestream Mach number. The upper surface of the airfoil very possibly may be an expansion surface. Since we will be concerned only with the Newtonian limits $\gamma \to \infty$ and $M_\infty \to \infty$, the upper surface will have very little effect and it will be ignored in what follows. Choose a coordinate system $O\bar{x}\bar{y}$ fixed in space such that O is at the apex of the airfoil (when it is stationary in the plane $\bar{y}=0$) and $O\bar{x}$ axis coincides with some fixed direction, and $O\bar{y}$ axis points downward.

Assuming an inviscid, nonheat conducting, perfect gas, the unsteady flow equations of continuity, momentum, and energy for the pressure $\bar{p}(\bar{x},\bar{y},\bar{t})$, density $\bar{\rho}(\bar{x},\bar{y},\bar{t})$, and the components $\bar{u}(\bar{x},\bar{y},\bar{t})$ and $\bar{v}(\bar{x},\bar{y},\bar{t})$ of the velocity vector $\bar{q}$ may be written as

$$\bar{\rho}_{\bar{t}} + \nabla\cdot(\bar{\rho}\bar{q}) = 0 \tag{1a}$$

$$\bar{q}_{\bar{t}} + \bar{q}\cdot\nabla\bar{q} + (1/\bar{\rho})\nabla\bar{p} = 0 \tag{1b}$$

$$(\bar{p}/\bar{\rho}^{\gamma})_{\bar{t}} + \bar{q}\cdot(\bar{p}/\bar{\rho}^{\gamma}) = 0 \tag{1c}$$

The subscripts above and in what follows, if not otherwise stated, denote partial derivatives; $\bar{t}$ is the time, and ∇ is the usual differential operator.

The boundary condition at the airfoil surface is that the normal component of relative velocity vanishes; that is,

$$F_{\bar{t}} + \bar{q}\cdot\nabla F = 0 \tag{2}$$

where

$$F(\bar{x},\bar{y},\bar{t}) = \bar{y} - \bar{F}(\bar{x},\bar{t}) = 0 \tag{3}$$

is the equation of the lower surface during its oscillatory motion. At the shock wave, we should have

$$[\bar{\rho}(S_{\bar{t}} + \bar{q}\cdot\nabla S)] = 0 \tag{4a}$$

$$[\bar{\rho}(S_{\bar{t}} + \bar{q}\cdot\nabla S)^2 + (\nabla S)^2\bar{p}] = 0 \tag{4b}$$

$$\left[(S_{\bar{t}} + \bar{q}\cdot\nabla S)^2 + \frac{2\gamma}{\gamma-1}(\nabla S)^2\frac{\bar{p}}{\bar{\rho}}\right] = 0 \tag{4c}$$

$$[\bar{q}\times\nabla S] = 0 \tag{4d}$$

where the square brackets denote the change in the enclosed quantity across the shock wave and

$$S(\bar{x},\bar{y},\bar{t}) = \bar{y} - \bar{S}(\bar{x},\bar{t}) = 0 \tag{5}$$

is the equation of the shock wave, as yet unknown and to be found as part of the solution. Equations (4a-4c) are the usual conservation equations of mass, momentum, and energy across an oblique shock wave, whereas Eq. (4d) is a vector equation equivalent to a scalar one expressing the conservation of the tangential velocity. Equations (4) are sufficient to find the functions $\bar{u}$, $\bar{v}$, $\bar{p}$, and $\bar{\rho}$ just behind the shock in terms of the freestream conditions and the (unknown) shock wave. Equations (1), (2), and (4) give the full problem, within the assumptions stated, that needs to be solved in order to find the unsteady flow past oscillating wings.

In what follows, we outline briefly the steady Newtonian flow theory given in Ref. 28, which we will extend and perturb in this section. Define a small parameter ϵ by

$$\epsilon = \frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_\infty^2}$$

Physically, ϵ is the density ratio across a normal shock wave, and it obviously approaches zero as $\gamma \to 1$ and $M_\infty \to \infty$. Regardless of the value of γ, ϵ is less than 1 for supersonic flow, equal to 1 for sonic flow, and greater than 1 for subsonic flow. Thus, ϵ can be used as another alternative (to M_∞) in classifying flowfields as being subsonic, sonic, or supersonic. Further, by combining the Newtonian limits $\gamma \to 1$ and $M_\infty \to \infty$ with a geometric limiting process in which the airfoil thickness approaches zero as $\epsilon \to 0$ while the angle of attack α remains fixed, very simple equations giving a correction to the Newtonian limits are found and solved easily.

We now extend the above theory to unsteady flow as follows. Let λ_0 be the amplitude of oscillation, which is assumed to be small. It can be shown that, when higher order terms in

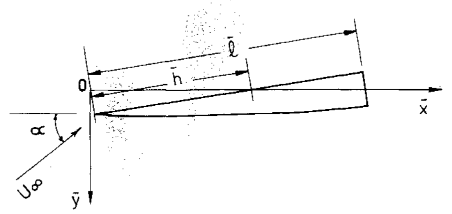

Fig. 1 Oscillating airfoil geometry and coordinate system.

λ_0 are neglected, the equation of the airfoil surface can be approximated by

$$\bar{F}(\bar{x},\bar{t}) = \bar{F}_0(\bar{x}) + \lambda_0 e^{i\omega\bar{t}}\,(\bar{h}-\bar{x}) + O(\lambda_0^2) \tag{6}$$

where $\bar{F}_0(\bar{x})$ is the equation of the lower surface when it is stationary in the plane $\bar{y}=0$, i is the imaginary unity (in what follows we take the real part of all complex functions), and ω is the circular frequency. Define new independent variables as

$$x = \bar{x}/\bar{\ell} \tag{7a}$$

$$y = \frac{\bar{y} - \lambda_0 e^{i\omega\bar{t}}\,(\bar{h}-\bar{x})}{\bar{\ell}\epsilon\,\tan\alpha} \tag{7b}$$

$$t = \bar{t}U_\infty/\bar{\ell} \tag{7c}$$

At the airfoil surface, we have

$$y = \frac{\bar{F}_0(\bar{x})}{\bar{\ell}\epsilon\,\tan\alpha} \equiv F_0(x)$$

Now, assume the following asymptotic expansions as a generalization of the steady flow to the oscillating flow case:

$$\frac{\bar{u}(\bar{x},\bar{y},\bar{t})}{U_\infty\,\cos\alpha} = 1 + \epsilon u(x,y) + \lambda_0 e^{ikt}\,U_1(x,y) + \cdots \tag{8a}$$

$$\frac{\bar{v}(\bar{x},\bar{y},\bar{t})}{U_\infty\,\sin\alpha} = \epsilon v(x,y) + \lambda_0 e^{ikt}\,V_0(x,y) + \lambda_0\epsilon e^{ikt}\,V_1(x,y) + \cdots \tag{8b}$$

$$\frac{\bar{p}(\bar{x},\bar{y},\bar{t}) - P_\infty}{\rho_\infty U_\infty^2\,\sin^2\alpha} = 1 + \epsilon p(x,y) + \lambda_0 e^{ikt}\,P_0(x,y) + \lambda_0\epsilon e^{ikt}\,P_1(x,y) + \cdots \tag{8c}$$

$$\frac{\rho_\infty}{a\bar{\rho}(\bar{x},\bar{y},\bar{t})} = \epsilon - \epsilon^2\rho(x,y) - \lambda_0\epsilon e^{ikt}\,R_1(x,y) + \cdots \tag{8d}$$

$$\frac{\bar{S}(\bar{x},\bar{t})}{\bar{\ell}\,\tan\alpha} = \epsilon F_0(x) + \epsilon S_0(x) + \lambda_0 e^{ikt}\,\cot\alpha(h-x) + \lambda_0 e^{ikt}\epsilon S_1(x) + \cdots \tag{8e}$$

where $k = \omega\bar{\ell}/U_\infty$ is the reduced frequency and a is a constant to be found. Substituting Eqs. (8) into Eqs. (1), (2), and (4) and retaining the lowest terms in ϵ and λ_0, we first get

$$a = \frac{1 + N\sin^2\alpha}{(1+N)\,\sin^2\alpha} \tag{9}$$

where

$$N = \frac{(\gamma-1)M_\infty^2}{2} \tag{10}$$

We also get

$$V_{0y} = 0 \tag{11}$$

$$V_0[x,F_0(x)] = V_0[x,F_0(x)+S_0(x)] = \frac{[ik(h-x) - \cos\alpha]}{\sin\alpha} \tag{12}$$

Equations (11) and (12) have the solution

$$V_0(x,y) \equiv V_0(x) = \frac{[ik(h-x) - \cos\alpha]}{\sin\alpha} \tag{13}$$

Then we get the following systems I–III of equations.

System I

$$v_y = 0 \tag{14a}$$

$$u_x + vu_y = 0 \tag{14b}$$

$$v_x + ap_y = 0 \tag{14c}$$

$$\left(\frac{\partial}{\partial x} + v\,\frac{\partial}{\partial y}\right)(p-\rho) = 0 \tag{14d}$$

at

$$y = F_0(x), \quad v = F_0'(x) \tag{15}$$

at

$$y = F_0(x) + S_0(x), \quad \rho = \frac{2[F_0'(x) + S_0'(x)]}{(1 + N\sin^2\alpha)} \tag{16a}$$

$$p = 2[F_0'(x) + S_0'(x)] - \frac{(\operatorname{cosec}^2\alpha + N)}{(1+N)} \tag{16b}$$

$$u = -\tan^2\alpha\,[F_0'(x) + S_0'(x)] \tag{16c}$$

$$v = F_0'(x) + S_0'(x) - \frac{(N + \operatorname{cosec}^2\alpha)}{(1+N)} \tag{16d}$$

System II

$$\left(ik + \cos\alpha\,\frac{\partial}{\partial x} + v\,\cos\alpha\,\frac{\partial}{\partial y}\right)R_1 + \cos\alpha(U_{1x} + V_{1y}) = -\cos\alpha\,\cot\alpha\,u_y \tag{17a}$$

$$\left(ik + \cos\alpha\,\frac{\partial}{\partial x} + v\,\cos\alpha\,\frac{\partial}{\partial y}\right)U_1 = 0 \tag{17b}$$

$$a\,\cos\alpha\,\frac{\partial P_0}{\partial y} = -ikV_0 - \cos\alpha\,\frac{\mathrm{d}V_0}{\mathrm{d}x} \tag{17c}$$

$$\left(ik + \cos\alpha\,\frac{\partial}{\partial x} + v\,\cos\alpha\,\frac{\partial}{\partial y}\right)(P_0 - R_1) = 0 \tag{17d}$$

at

$$y = F_0(x), \quad V_1 = F_0'(x)U_1 - \cot\alpha\,u \tag{18}$$

at

$$y = F_0(x) + S_0(x), \quad U_1 = \tan\alpha \tag{19a}$$

$$R_1 = \frac{2[ik(h-x) - \cos\alpha]}{\sin\alpha(1 + N\sin^2\alpha)} \tag{19b}$$

$$P_0 = \frac{2[ik(h-x) - \cos\alpha]}{\sin\alpha} \tag{19c}$$

$$V_1 = \frac{ik}{\cos\alpha}\,S_1(x) + S_1'(x) + 2[a + F_0'(x)] + \frac{(\operatorname{cosec}^2\alpha - N)[ik(h-x) - \cos\alpha]}{\sin\alpha(1+N)} \tag{19d}$$

System III

$$-a\,\tan\alpha\,P_{1y} = ik\,\sec\alpha V_1 + V_{1x} + v\,\tan\alpha V_{1y} + (\rho + u)V_{0x} + ik\rho\,\sec\alpha V_0 + v_x(R_1 + U_1) \tag{20}$$

at

$$y = F_0(x) + S_0(x)$$

$$P_1 = 2\left\{S_1'(x) + \frac{ik}{\cos\alpha} S_1(x) + \frac{[F_0'(x) + S_0'(x)]\ [ik(h-x) - \cos\alpha]}{\sin\alpha}\right\}$$

$$+ 2\tan\alpha[F_0'(x) + S_0'(x)] - \frac{2N[ik(h-x) - \cos\alpha]}{(1+N)\sin\alpha} \tag{21}$$

System I is very simple and identical to the one given in Ref. 28 [$F_0(x)$ is denoted by $F(x)$ in Ref. 28]. Systems II and III are linear and give the unsteady flow past pitching oscillating pointed-nose, two-dimensional thin airfoils at moderate or large angles of attack, provided that the bow shock is attached to the wing apex. System III is included in order to get a first-order theory showing the effects of γ and M_∞. The solution of system I is straightforward and was given in Ref. 28 in the following form:

$$v(x,y) = F_0'(x), \qquad S_0(x) = ax \tag{22a}$$

$$u(x,y) = -\tan^2\alpha\left\{a + F_0'\left[\frac{y - F_0(x)}{a}\right]\right\} \tag{22b}$$

$$p(x,y) = -(1/a)F_0''(x)\ [y - F_0(x) - ax] + 2F_0'(x) + a \tag{22c}$$

As was mentioned in Ref. 28, a basic assumption in deriving system I is that α remains fixed as $\epsilon \to 0$. Therefore, system I is expected to give good results only for moderate or large α and a nonuniformity should be expected if α and ϵ have the same (small) order of magnitude. The function ρ was not found in Ref. 28. Since it will be needed for the unsteady flow solution, we find it in what follows. The general solution of Eq. (14d) is given by

$$\rho = p + \phi[y - F_0(x)]$$

where ϕ is an arbitrary function to be fixed by the boundary conditions [Eqs. (16a) and (16b)]. Thus, ρ is given by

$$\rho(x,y) = -(1/a)F_0''(x)\ [y - F_0(x) - ax] + 2F_0'(x) - \frac{2N}{a(1+N)} F_0'\left(\frac{y - F_0(x)}{a}\right) + \frac{2}{\sin^2\alpha(1+N)} \tag{23}$$

III. Solutions for Unsteady Flow

The function P_0 could easily be found from Eqs. (17c) and (19c) and is given by

$$P_0(x,y) = \frac{2}{a\sin 2\alpha}\ [2ik\cos\alpha + k^2(h-x)]\ [y - F_0(x) - ax] + \frac{2}{\sin\alpha}\ [ik(h-x) - \cos\alpha] \tag{24}$$

Further, the general solution of Eq. (17b) is given by

$$\ell n\, U_1 = \psi[y - F_0(x)] - ikx\sec\alpha$$

where ψ is an arbitrary function. Using Eq. (19a), ψ can be found and thus we get

$$\ell n\left(\frac{U_1}{\tan\alpha}\right) = \frac{ik\sec\alpha}{a}\ [y - F_0(x) - ax] \tag{25}$$

For the purpose of finding the aerodynamic derivative, the function R_1 is not needed, and it can be eliminated from Eqs. (17a) and (17d). Continuing with the preceding exact solutions for P_0 and U_1 may prove to be tiresome. Therefore, in what follows, we will restrict ourselves to small values of k only by neglecting terms of order $O(k^2)$ and higher. Equations (24) and (25) thus become

$$P_0(x,y) = \frac{2ik}{a\sin\alpha}\ [y - F_0(x) - ax] + \frac{2}{\sin\alpha}\ [ik(h-x) - \cos\alpha] + O(k^2) \tag{26a}$$

$$U_1(x,y) = \tan\alpha + \frac{ik\tan\alpha}{a\cos\alpha}\ [y - F_0(x) - ax] + O(k^2) \tag{26b}$$

Now, inserting Eqs. (17d), (26a), and (26b) into Eq. (17a) and then using the boundary condition of Eq. (18), we get V_1 as

$$V_1(x,y) = ik[y - F_0(x)]\left[\frac{6}{\sin\alpha} + \frac{\tan\alpha}{\cos\alpha} + \frac{\tan\alpha\, F_0'(x)}{a\cos\alpha}\right] + \tan\alpha\left[a + F_0'\left(\frac{y - F_0(x)}{a}\right)\right] + \tan\alpha\, F_0'(x)\left(1 - \frac{ikx}{\cos\alpha}\right) + O(k^2) \tag{27}$$

This completes the solution of system II. We now proceed to find the surface pressure $P_1[x, F_0(x)]$ form system III, since, in general, the function P_1 is not needed. Also, the shock wave function S_1 is not needed and can be eliminated from Eqs. (19d) and (21). The result is given by

$$P_1[x, ax + F_0(x)] = 2V_1[x, ax + F_0(x)] + 2(\tan\alpha - 2)\ [a + F_0'(x)] + \frac{2}{\sin\alpha}\ [ik(h-x) - \cos\alpha] \times \left[a + F_0'(x) - \frac{1}{(1+N)\sin^2\alpha}\right] \tag{28}$$

Equation (28) shows that P_1 at the shock wave is independent of S_1. Now, integrating Eq. (20) from the body [$y = F_0(x)$] to the shock [$y = ax + F_0(x)$] and using Eq. (28), we get

$$P_1[x, F_0(x)] = P_1[x, ax + F_0(x)] + x\cot\alpha \int_0^1 [ik\sec\alpha\, V_1 + V_{1x} - ik\,\mathrm{cosec}\alpha(2\rho + u) + F_0''(x)\ (R_1 + U_1) + \tan\alpha\, F_0'(x) V_{1y}]\ d\xi \tag{29}$$

where ξ is defined by

$$y = ax\xi + F_0(x) \tag{30}$$

and

$$V_1 = V_1(x, \xi), \quad u = u(x, \xi), \ldots, \text{etc.}$$

The integrals in Eq. (29) can be found in closed form. If we write

$$P_1[x, F_0(x)] = P_{1r} + ikP_{1i}$$

then,

$$P_{1r} = (6\tan\alpha - 2\cot\alpha - 4)\ [a + F_0'(x)] + \left(\frac{\tan\alpha - 1}{a}\right) F_0'(x)\ [F_0'(x) - F_0'(0)] + 2\left[1 - \cot^2\alpha + \frac{N\cot^2\alpha}{a(1+N)}\right] x F_0''(x) + \frac{4(\cot^2\alpha - 1 - N)}{\sin 2\alpha(1+N)} \tag{31a}$$

$$
\begin{aligned}
P_{1i} = {} & 2ax\left(\frac{6}{\sin\alpha}+\frac{\tan\alpha}{\cos\alpha}\right)+\frac{2(h-x)}{\sin\alpha}\left[F_0'(x)+\frac{N}{1+N}\right] \\
& + x\sec\alpha\left[a-\frac{1}{2}xF_0''(x)+\frac{F_0(x)}{x}\right] \\
& + (1-\cot\alpha)xF_0'(x)\left[\frac{6}{\sin\alpha}+\frac{\tan\alpha}{\cos\alpha}+\frac{\tan\alpha\, F_0'(x)}{a\cos\alpha}\right] \\
& + x\cot\alpha\, F_0''(x)\left[-x\left(\frac{2\cos\alpha+\sin\alpha\tan\alpha}{\sin 2\alpha}\right)\right. \\
& \left. + \frac{2a(1+N)(h-x)-2Nh}{a\sin\alpha(1+N)}\right] - x\operatorname{cosec}\alpha\cot\alpha\left\{xF_0''(x)\right. \\
& + 4F_0'(x)-\left[\frac{4N}{a(1+N)}+\tan^2\alpha\right]\frac{F_0(x)}{x} \\
& \left. + 4a - a\tan^2\alpha - \frac{4N}{1+N}\right\}
\end{aligned} \tag{31b}
$$

Having found the unsteady surface pressure, we now find the stability derivative in the following section.

IV. Stability Derivative and Results

With the simple solution above for the unsteady surface pressure, we derive formulas for the stability derivative as follows. First, the pitching moment coefficient C_m is given by

$$
\begin{aligned}
C_m = \frac{M_h}{\frac{1}{2}\rho_\infty U_\infty^2 \bar{\ell}^3} = 2\int_0^{\bar{\ell}} \frac{1}{\rho_\infty U_\infty^2}\left\{\bar{p}[\bar{x},\bar{F}(\bar{x},\bar{t}),\bar{t}]\right. \\
\left. -\bar{p}_0[\bar{x},\bar{F}_0(\bar{x})]_\ell\right\}(x-h)\,\mathrm{d}x
\end{aligned} \tag{32}
$$

where M_h denotes the pitching moment about an axis at a distance $\bar{h}$ for the vertex, ℓ the lower surface of the airfoil, and $\bar{p}_0$ the steady physical pressure at airfoil surface when it is stationary in the plane $\bar{y}=0$. As mentioned before, the upper surface will be an expansion surface at moderate and large angles of attack and have negligible effect in the Newtonian limit. Since

$$C_m = \lambda_0 e^{ikt}\,(C_{m_\theta} + ik\,C_{m_{\dot\theta}})$$

where $-C_{m_\theta}$ and $-C_{m_{\dot\theta}}$ are, respectively, the in-phase and the out-of-phase components of the stability derivative, we get the following formulas for $-C_{m_\theta}$ and $-C_{m_{\dot\theta}}$

$$-C_{m_\theta} = 2\sin^2\alpha\int_0^1 (P_{0r}+\epsilon P_{1r})(h-x)\,\mathrm{d}x \tag{33a}$$

$$-C_{m_{\dot\theta}} = 2\sin^2\alpha\int_0^1 (P_{0i}+\epsilon P_{1i})(h-x)\,\mathrm{d}x \tag{33b}$$

where

$$P_{0r} = -2\cot\alpha, \qquad P_{0i} = \frac{2(h-2x)}{\sin\alpha}$$

Using numerical quadrature, the above integrals can easily be found. We now can investigate the stability of the pitching oscillations by drawing curves as follows. In Fig. 2, the variation of $-C_{m_\theta}$ with h is shown for $\alpha=25$ and 60 deg for the concave exponential airfoil $\bar{F}_0(\bar{x})=\bar{\ell}\,(e^{0.2x}-1)$ at $\gamma=1.1$ and $M_\infty=6$. The curves show that increasing α has large increasing effect on $-C_{m_\theta}$ for $h<0.53$ and large decreasing effect for $h>0.53$. Figure 3 shows the variation of $-C_{m_\theta}$ with h for the same airfoil at the same γ, M_∞, and α as in Fig. 2. It shows that $-C_{m_{\dot\theta}}>0$ for all h when $\alpha=60$ deg; however, for $\alpha=25$ deg, $-C_{m_{\dot\theta}}$ is <0 for a certain range of pivot positions around $h=1.0$. This shows that, at moderate angles of attack, the pitching oscillations may become dynamically unstable.

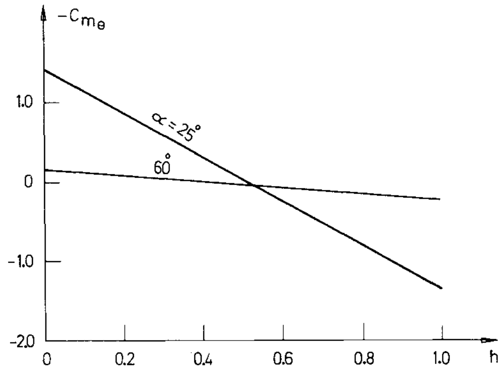

Fig. 2 Variation of $-C_{m_\theta}$ with h for the concave airfoil [$\bar{F}_0(\bar{x})=\bar{\ell}$ ($e^{0.2x}-1$), $\gamma=1.1$, $M_\infty=6$].

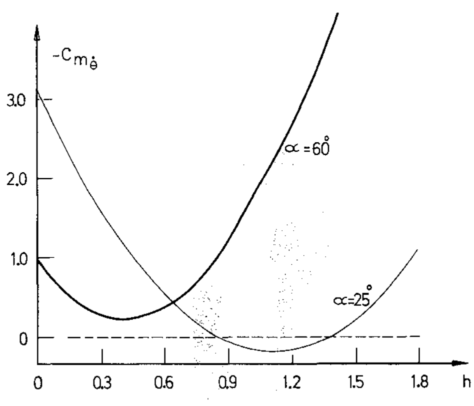

Fig. 3 Variation of $-C_{m_{\dot\theta}}$ with h for the concave airfoil [$\bar{F}_0(\bar{x})=\bar{\ell}$ ($e^{0.2x}-1$), $\gamma=1.1$, $M_\infty=6$].

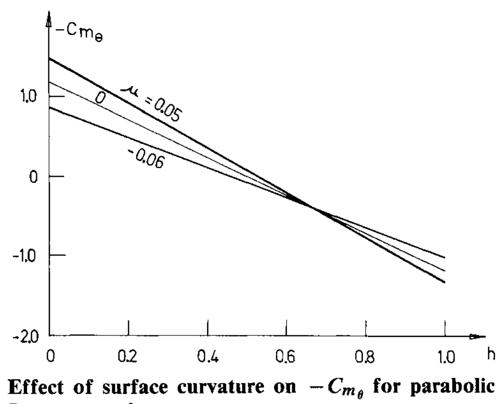

Fig. 4 Effect of surface curvature on $-C_{m_\theta}$ for parabolic airfoils [$\bar{F}_0(\bar{x})=\bar{\ell}$ ($0.15x+\mu x^2$), $\gamma=1.1$, $M_\infty=6$, $\alpha=25$ deg].

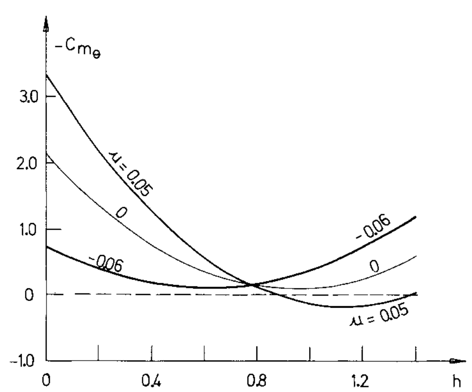

Fig. 5 Effect of surface curvature on $-C_{m_{\dot\theta}}$ for parabolic airfoils [$\bar{F}_0(\bar{x})=\bar{\ell}$ ($0.15x+\mu x^2$), $\gamma=1.1$, $M_\infty=6$, $\alpha=25$ deg].

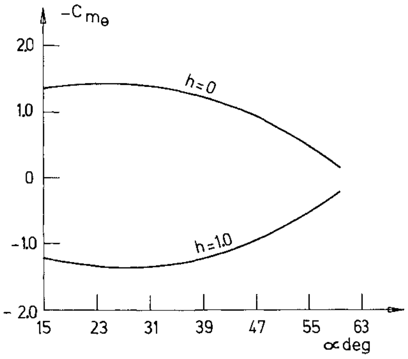

Fig. 6 Variation of $-C_{m_\theta}$ with α for the concave airfoil $[\bar{F}_0(\bar{x})=\bar{\ell}\,(e^{0.2x}-1),\ \gamma=1.1,\ M_\infty=6]$.

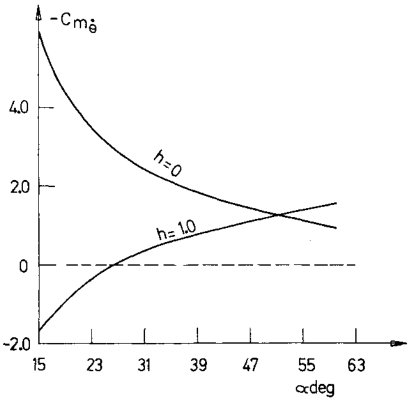

Fig. 7 Variation of $-C_{m_{\dot\theta}}$ with α for the concave airfoil $[\bar{F}_0(\bar{x})=\bar{\ell}\,(e^{0.2x}-1),\ \gamma=1.1,\ M_\infty=6]$.

Figure 4 shows the effect of surface curvature on $-C_{m_\theta}$ for a class of parabolic airfoils given by $\bar{F}_0(\bar{x})=\bar{\ell}\,(0.15x+\mu x^2)$, with $\mu=-0.06$ (convex surface), 0 (flat plate), and 0.05 (concave surface) at $\gamma=1.1$, $M_\infty=6.0$, and $\alpha=25$ deg. It is shown that for $h<0.68$, increasing curvature from convex to concave increases $-C_{m_\theta}$, while for $h>0.68$ convex curvature increases $-C_{m_\theta}$.

Figure 5 shows the variation of $-C_{m_{\dot\theta}}$ with h for the same class of airfoils considered in Fig. 4 at the same γ, M_∞, and α. It shows that, for the convex and the flat-plate airfoil, the damping derivative is >0 for all h, but for the concave airfoil it becomes <0 for certain values of h (around 1.2). Thus, as deduced before, at moderate angles of attack, the pitching oscillations may become dynamically unstable and the concave curvature tends to decrease the stability for certain pivot positions (given by $h>$ about 0.8 in this case) or even to make the airfoil dynamically unstable. On the other hand, for other pivot positions ($h<0.8$ in this case), concave curvature increases the stability of the airfoil.

Figures 6 and 7 show the variation of $-C_{m_\theta}$ and $-C_{m_{\dot\theta}}$ with α for $h=0$ and 1.0 for the concave airfoil $\bar{F}_0(\bar{x})=\bar{\ell}\,(e^{0.2x}-1)$ at $\gamma=1.1$ and $M_\infty=6$. The two figures show that the angle of attack has a large effect on the stability derivative. Figure 8 shows the stability boundary for the airfoil $\bar{F}_0(\bar{x})=\bar{\ell}\,(e^{0.15x}-1)$, for two cases: 1) $\gamma=1.4$, $M_\infty=5$, and 2) $\gamma=1.05$, $M_\infty=10$. The figure shows that there is a certain region of moderate angles of attack for which the pitching oscillations are unstable and that this instability region enlarges (case 2) as we approach the Newtonian limit. It is interesting to observe that by increasing the angle of attack further (to about 70 deg), we again get another region of instability (not shown on figure). But, for such a large incidence, the shock wave becomes detached and the present analysis no longer holds.

Although one would expect the existence of a few testing results in space programs in the United States and other countries, as well as a few reports of experimental work in ballis-

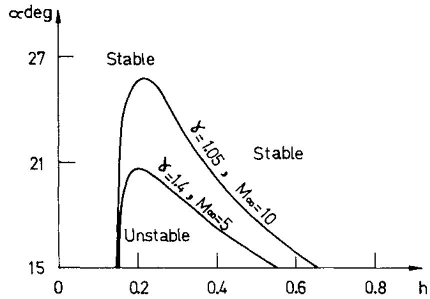

Fig. 8 Stability boundary for the concave airfoil $[\bar{F}_0(\bar{x})=\bar{\ell}\,(e^{0.15x}-1)]$.

tics, the author could not find such reports to compare with the present theory. Finally, it should be mentioned that viscous forces, which are neglected here, have considerable effect in hypersonic flow, particularly at moderate or large angles of attack.

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